

8.5 Large Sample Approx.

Thm: $Y_v \sim \chi^2(v)$, then

$$Z_v = \frac{Y_v - v}{\sqrt{2v}} \xrightarrow{d} Z \sim N(0,1)$$

$$\frac{\sum X_i - nu}{\sqrt{n} \sigma} \xrightarrow{d} N(0,1)$$

$$\begin{aligned} \mu &= E(X_i) \\ \sigma^2 &= V(X_i) \end{aligned}$$

If $X_i \sim \chi^2(1)$

$$Y_v = \sum_{i=1}^v X_i \sim \chi^2(v)$$

$$E(X_i) = 1$$

$$V(X_i) = 2 \Rightarrow \sigma = \sqrt{2}$$

Thus

$$\frac{Y_v - v}{\sqrt{2v}} = \frac{\sum X_i - v(1)}{\sqrt{v} \cdot \sqrt{2}} \xrightarrow{d} N(0,1)$$

$$\gamma = P\left[\chi^2_{\nu} \leq \underbrace{\chi^2_{\gamma}(\nu)}_{\text{percentile}}\right] = \Phi\left(\frac{\underbrace{\chi^2_{\gamma}(\nu) - \nu}_{z_{\gamma}}}{\sqrt{2\nu}}\right)$$

$$\Rightarrow z_{\gamma} = \frac{\chi^2_{\gamma}(\nu) - \nu}{\sqrt{2\nu}}$$

$$\boxed{\chi^2_{\gamma}(\nu) \doteq \nu + z_{\gamma}\sqrt{2\nu}}$$

Ex: $\nu = 30$ $\gamma = .95$

$$\chi^2_{.95}(30) \doteq 30 + 1.645(\sqrt{60})$$

$$= 42.74$$

$$\chi^2_{.95}(30) = 43.77$$

A more accurate approx[↓]

$$\chi^2_{\gamma}(\nu) \doteq \nu \left[1 - \frac{2}{9\nu} + z_{\gamma} \sqrt{\frac{2}{9\nu}} \right]^3$$

$$\chi^2_{.95}(30) = 30 \left[1 - \frac{2}{9(30)} + 1.645 \sqrt{\frac{2}{9(30)}} \right]^3$$

$$= 43.768.$$

Wilson-Hilferty Approx.

ex: $V_n = \frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2(n-1)$

$$\frac{V_n - (n-1)}{\sqrt{2(n-1)}} \xrightarrow{d} Z \sim N(0, 1).$$

$$\begin{aligned} \frac{\frac{(n-1)S_n^2}{\sigma^2} - n + 1}{\sqrt{2(n-1)}} &= \frac{\frac{(n-1)S^2}{\sigma^2} - \frac{(n-1)\sigma^2}{\sigma^2}}{\sqrt{2(n-1)}} = \frac{\frac{(n-1)}{\sigma^2} (S^2 - \sigma^2)}{\sqrt{2} \sqrt{n-1}} \\ &= \frac{\sqrt{n-1} (S_n^2 - \sigma^2)}{\sqrt{2} \sigma^2} \xrightarrow{d} Z \sim N(0, 1) \end{aligned}$$

So, for large V ,

$$S_n^2 \sim N\left(\sigma^2, \frac{2\sigma^4}{n-1}\right)$$

Use Delta Method (p.77)

$$S_n \sim N\left(\sigma, \frac{\sigma^2}{2(n-1)}\right)$$

EX: t has a limiting normal dist.

$$E\left[\frac{\chi_v^2}{v}\right] = \frac{1}{v} E(\chi_v^2) = \frac{v}{v} = 1$$

$$V\left[\frac{\chi_v^2}{v}\right] = \frac{1}{v^2} V(\chi_v^2) = \frac{1}{v^2} 2v = \frac{2}{v}$$

Chebyshev's Inequality.

$$P\left[\left|\frac{\chi_v^2}{v} - 1\right| < \varepsilon\right] \geq 1 - \frac{2}{v\varepsilon^2} \xrightarrow{v \rightarrow \infty} 1$$

$$\frac{\chi_v^2}{v} \xrightarrow{P} 1$$

$$T_v = \frac{z}{\sqrt{\frac{\chi_v^2}{v}}}$$

$$\sqrt{\frac{\chi_v^2}{v}} \xrightarrow{P} \sqrt{1} = 1$$

$$T_v = \frac{z}{\sqrt{\frac{\chi_v^2}{v}}} \xrightarrow{d} \frac{z}{1} \sim N(0,1)$$

Slutsky's Thm, pt 3.

$$a_n X_n \rightarrow aX$$

$$a_n \xrightarrow{P} a \quad X_n \xrightarrow{d} X$$