

## 8.2 Statistics.

DEF: A function of observable R.V.

$$T = t(X_1, \dots, X_n)$$

which does not depend on any unknown parameters  
is called a statistic

EX: Sample mean.

$$\bar{X} = t(X_1, X_2, \dots, X_n) = \frac{1}{n} \sum X_i$$

Thm: If  $X_1, \dots, X_n$  denotes  
a R.V. from  $f(x)$ .

with  $E(X) = \mu$   
 $V(X) = \sigma^2$ , then

$$E(\bar{X}) = \mu$$

$$V(\bar{X}) = \frac{\sigma^2}{n}$$

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A statistic for which  
 $E(\text{stat}) = \text{parameter}$ ,

We say the stat. is an unbiased  
estimate of the parameter

Ex:  $Y \sim \text{HYP}(n, M, N)$

Suppose we want to estimate  $\frac{M}{N} = P$ .

Then

$$E\left(\frac{Y}{n}\right) = \frac{1}{n} E(Y) = \frac{1}{n} \frac{nM}{N} = \frac{M}{N}$$

$\frac{Y}{n}$  is an unbiased est. of  $\frac{M}{N}$

$$\begin{aligned} \text{Var}\left(\frac{Y}{n}\right) &= \frac{1}{n^2} \text{Var}(Y) = \frac{1}{n^2} n \frac{M}{N} \left(1 - \frac{M}{N}\right) \left(\frac{N-n}{N-1}\right) \\ &= \frac{M}{N} \left(1 - \frac{M}{N}\right) \left(\frac{1}{N-1}\right) \left(\frac{N}{n} - 1\right) \end{aligned}$$

$\text{Va}\left(\frac{Y}{n}\right) \rightarrow 0$  as  $n \rightarrow \infty$ .

Ex: Sample Variance.

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

$$= \frac{\sum_{i=1}^n X_i^2 - n\bar{X}^2}{n-1}$$

$S^2$  is a function of the random sample.