$\boxed{4.2}$ Cont.

Red X_1 (rows), X_2 White (columns)

$f(x_1,x_2)$	0	1	2	3	4
0	[4] .0016	[3] .0127	[2] .0383	[1] .0511	[0] .0254
1	[3] .0127	[2] .0769	[1] .1541	[0] .1022	0
2	[2] .0383	[1] .1541	[0] .1539	0	0
3	[1] .0511	[0] .1022	0	0	0
4	[0] .0254	0	0	0	0

$$f(x_1,x_2) = \frac{\binom{400}{x_1}\binom{400}{x_2}\binom{200}{4-x_1-x_2}}{\binom{1000}{4}}$$

What is the P(1 white, 1 red, 2 pink) = f(1,1) = .0769

What is prob of 1 white?

P(1 white) = P(1 white & 0 red) + P(1 white & 1 red) + ..

... + P(1 white & 4 red)

= f(0,1) + f(1,1) + f(2,1) + f(3,1) + f(4,1)

= .0127 + .0769 + .1541 + .1022 + 0

= .3459

Note that this is a probability without any regard to the red seeds. Such probabilities are called "marginal probabilities". To find these, you sum up over all the probabilities.

DEF:

If (x_1, x_2) of discrete R.V. has the joint pdf $f(x_1, x_2)$ then the marginal pdf of x_1 & x_2 are

$$f_1(x_1) = \sum_{x_2} f(x_1, x_2)$$

$$f_2(x_2) = \sum_{x_1} f(x_1, x_2)$$

Ex The marginal pdf of red seeds is

x_1	0	1	2	3	4
$f_1(x_1)$	.1291	3459	.3462	.1534	0254

To get the marginal for white seeds, add the columns down. You'll get the same answer as the red:

x_2	0	1	2	3	4
$f_2(x_2)$	.1291	3549	.3462	.1534	.0254

To get the marginal for pink seeds:

You're looking for constant values of pink

1) Make a new table where pink is one of the variables besides white or red and then follow the example of before

2) Just try to do it from the current table

Joint CDF

DEF The joint CDF of k random vars $X_1, X_2, \cdots, X_k$ is the function defined by

$$F(x_1, x_2, \quad, x_k) = P[X_1 \le x_1, X_2 \le x_2, \quad, X_k \le x_k]$$

Thm 4.22
A function $F(x_1, x_2)$ is a bivariate CDF iff

$$\lim_{x_1 \to -\infty} F(x_1, x_2) = F(-\infty, x_2) = 0 \text{ for all } x_2$$

$$\lim_{x_2 \to -\infty} F(x_1, x_2) = F(x_1, -\infty) = 0 \text{ for all } x_1$$

$$\lim_{\substack{x_1 \to \infty \\ x_2 \to \infty}} F(x_1, x_2) = F(\infty, \infty) = 1$$

$$F(b, d) - F(b, c) - F(a, d) + F(a, c) \ge 0$$

for all $a < b$ and $c < d$ (Monotonicity)

and

$$\lim_{h \to 0^+} F(x_1 + h, x_2) = \lim_{h \to 0^+} F(x_1, x_2 + h) = F(x_1, x_2)$$

for all x_1, x_2 (Right Continuous)

$\boxed{4.3}$ Joint Cont. Dist

DEF: k dimensional RV $X = (X_1, X_2, \cdots, X_k)$
$f(x_1, x_2, \cdot \ , x_k)$ joint pdf
The joint CDF is written as

$$F(x_1, x_2, \cdots, x_k) = \int_{-\infty}^{x_k} \cdots \int_{-\infty}^{x_{k-1}} \cdots \int_{-\infty}^{x_1} f(t_1, t_2, \quad, t_k)\, dt_1 dt_2 \quad dt_k$$

for all $x = (x_1, \quad, x_k)$

To get the joint pdf, then

$$f(x_1, x_2, \cdots, x_k) = \frac{\partial^k}{\partial x_1 \partial x_2 \cdots \partial x_k} F(x_1, x_2, \cdots, x_k)$$

$\boxed{\text{Thm } 4.3.1}$ Conditions of a joint pdf.

$$f(x_1, x_2, \cdots, x_k) \geq 0 \quad \text{for all } x$$

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1, x_2, \cdots, x_k)\, dx_k \cdots dx_1 = 1$$

Example Let X_1 denote the concentration of a certain substance in one trial of an experiment, and X_2 the concentration of the substance on a second trial of an experiment.

Suppose $f(x_1, x_2) = 4x_1 x_2 I_{(0,1)}(x_1) I_{(0,1)}(x_2)$

The joint CDF is (for $0<x_1<1$ and $0<x_2<1$

$$F(x_1, x_2) = \int_{-\infty}^{x_2} \int_{-\infty}^{x_1} f(t_1, t_2)\, dt_1\, dt_2$$

$$= \int_0^{x_2} \int_0^{x_1} 4t_1 t_2 \, dt_1\, dt_2$$

$$= x_1^2 x_2^2, \quad 0<x_1<1, \; 0<x_2<1$$

If $x_1 > 1$ and $0 < x_2 < 1$, then

$$F(x_1, x_2) = x_2^2$$

If $x_2 > 1$ and $x_1 > 1$, then

$$F(x_1, x_2) = 1$$

If $x_2 > 1$ and $0 < x_1 < 1$, then

$$F(x_1, x_2) = x_1^2$$

$$F(x_1, x_2) = \begin{cases} 1, & x_1 > 1,\ x_2 > 1 \\ x_1^2, & x_2 > 1,\ 0 < x_1 < 1 \\ x_2^2, & x_1 > 1,\ 0 < x_2 < 1 \\ x_1^2 x_2^2, & 0 < x_1 < 1,\ 0 < x_2 < 1 \end{cases}$$

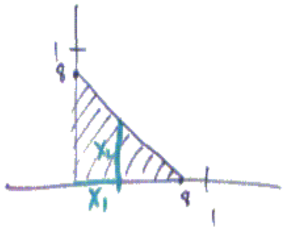
It is possible to evaluate joint prob by integrating over the appropriate region

$$P[X \in A] = \int \cdots \int_A f(x_1, x_2, \dots, x_k)\, dx_1 \cdots dx_k$$

Ex: Suppose we want to find the prob that the average concentration is less than .4

$$\frac{x_1 + x_2}{2} = 4$$

$$= x_1 + x_2 = 8 \Rightarrow \boxed{x_2 = -x_1 + 8}$$



$$P\left(\frac{x_1 + x_2}{2} < 4\right) = \int_0^{4/5} \int_0^{-x_2 + \frac{4}{5}} 4 x_1 x_2 \, dx_1 \, dx_2$$

$$= \frac{128}{1875}$$

(an good Math214 student can integrate this!)

DEF 4.3.2

If the pair (X_1, X_2) of Cont R.V. has the joint pdf $f(x_1, x_2)$, then the marginal pdf's of X_1 & X_2 are

$$f_1(x_1) = \int_{-\infty}^{\infty} f(x_1, x_2)\, dx_2$$

$$f_2(x_2) = \int_{-\infty}^{\infty} f(x_1, x_2)\, dx_1$$

DEF 4.3.3.

$X = (X_1, \cdots, X_k)$ k dimensional R.V.

$F(x_1, \cdots, x_k)$ joint CDF. Then the marginal CDF is

$$F_j(x_j) = \lim_{\substack{x_i \to \infty \\ i \neq j}} F(x_1, \cdots, x_k)$$

The marginal pdf is

$$f_j(x_j) = \begin{cases} \displaystyle\sum \cdot \sum_{i \neq j} f(x_1, \cdots, x_j, \cdots, x_k) \\ \displaystyle\int \cdots \int_{i \neq j} f(x_1, \cdots, x_j, \cdots, x_k)\, d \cdots d \end{cases}$$

Ex. Let X_1, X_2, and X_3 be cont w/ joint pdf the form

$$f(x_1,x_2,x_3) = \begin{cases} c, & 0<x_1<x_2<x_3<1 \\ 0, & \text{otherwise} \end{cases}$$

Note. $c=6$ why? How?

Suppose we want the marginal of X_3

From $f_j(x_j)$ above,

$$f_3(x_3) = \int_0^{x_3}\int_0^{x_2} 6\,dx_1\,dx_2 \overset{\text{integration}}{\underset{\text{magic}}{=}} \begin{cases} 3x_3^2, & \text{if } 0<x_3<1 \\ 0, & \text{otherwise} \end{cases}$$

Find the joint pdf of X_1 & X_2

$$f(x_1,x_2) = \int_{-\infty}^{\infty} f(x_1,x_2,x_3)\,dx_3$$

$$= \int_{x_2}^{1} 6\,dx_3 = 6x_3\Big|_{x_2}^{1}$$

$$= 6(1-x_2)$$

$$f(x_1,x_2) = \begin{cases} 6(1-x_2), & 0<x_1<x_2<1 \\ 0, & \text{otherwise} \end{cases}$$