

2.1 Intro to Random Variables (R.V)

DEF 2.1.1 A random variable, say X , is a function defined over a sample space S , that associates a real number

$X(e) = x$, with each possible outcome e in S .
↑ capital ↑ little

Capital letters indicate random variables (X, Y, Z)

lowercase letters indicate possible values the R.V can attain.

EX: Flip a coin 3 times

Sample space: $S = \{$ HHH,
THH
HTH
HHT
TTH
THT
HTT
TTT $\}$

$X = \#$ of heads in 3 tosses.

$$X(\text{HHH}) = 3 \quad X(\text{TTT}) = 1$$

$$X(\text{HTH}) = 2 \quad X(\text{TTT}) = 0$$

EX: Roll 2 dice

1	2	3	4	5	6
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

X = maximum of 2 dice $\rightarrow X(5,4) = 5$

Y = sum of two dice $X(5,4) = 9$

Z = min of the 2 dice

2.2 Discrete R.V.

DEF: If the set of all possible values of a R.V. X , is a countable set (Note: could be infinite)

x_1, x_2, \dots, x_n or x_1, x_2, \dots

then X is called a discrete R.V. The function

$$f(x) = P[X=x], \quad x = x_1, x_2, \dots$$

small big small

that assigns prob. to each possible x will be called the discrete prob. density function (pdf) (also called pmf = prob. mass function)

Sometimes we write $f_X(x)$ instead of $f(x)$

Thm 2.2.1

A function $f(x)$ is a discrete pdf iff it satisfies both of the following for at most a countably infinite set of real #'s

1) $f(x_i) \geq 0$ for all x_i

2) $\sum_{\text{all } x_i} f(x_i) = 1$

proof. follows from axioms of prob.

Homework!

ex: Flip a coin 3 times.

$$S = \{HHH, TTH, THT, HTT, TTH, HTH, HHT, TTT\}$$

$X = \#$ of Heads in 3 tosses

x	$f(x)$
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

x	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

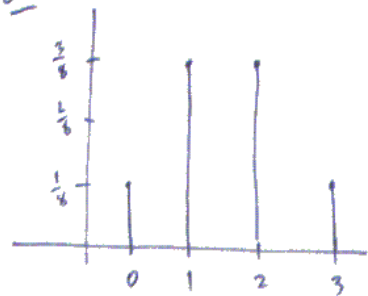
ex: $X = \text{Max of two dice}$

x	$f(x)$
1	
2	
3	
4	
5	
6	

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Graphical Representation

Ex:



$$f(x) = {}_3C_x (5)^3$$

formulas:

You can also write $f(x)$
as a formula.

$$f(x) = \frac{2x-1}{36}, x=1, 6$$

