1.1

Deterministic models

- describes a mathematical model to predict the observed value of some characteristic.

$$v = gt \quad , \text{ where } g = 9.8 \, m/s^2$$

For such a model, carrying out repeated experiments under ideal cond would result with the same velocity each time, and it is predicted by the model

We are looking at stochastic models.

Random instead of deterministic

- # of particles emitted by a radioactive substance
- time until failure of a component.
- game of chance

1.2 Notation & Terminology.

<u>Experiment</u> - process of obtaining an observed result of some phenomenon.

A performance of a experiment is a <u>trial</u>.

An observed outcome of a trial is called an <u>outcome</u>

The set of all possible outcomes is called the <u>sample space</u>

<u>Ex</u>: Flip a coin twice.

$S = \{HH, TH, HT, TT\}$

Ex: Suppose we count the number of heads.

$S = \{0, 1, 2\}$

<u>Ex</u>: Toss a coin repeatedly until a head shows.

$S = \{H, TH, TTH, TTTH, \cdots\}$

EX: A light bulb is tested to see how long it lasts:

$$S = \{ t \mid 0 \le t < \infty \}$$

A sample space is <u>finite</u> if it consists of a finite number of outcomes.

$$S = \{ e_1, e_2, \cdots, e_N \}. \quad N < \infty.$$

<u>Countably infinite</u> if the outcomes can be placed into a 1-1 correspondence with the natural numbers.

<u>DEF</u>: If a sample space is either finite or countably infinite, then it is called a <u>discrete</u> sample space

A set that is finite or countably infinite is said to be <u>countable</u>.

A sample space that involves uncountably many outcomes is called a <u>continuous sample space</u>

An <u>event</u> is a subset of the sample space

If A is an event, A occurs if it contains the outcome that occurred

Set Notation

Unions → "or"

Intersections → "and"

Ex: A, B sets.

$A \cap B$ ⇒ event in A and in B

intersections

$A \cup B$ ⇒ event is in A or in B.

Union.

A' or $\bar{A}$ ⇒ complement of A. The event does not belong to A.

A but not B = $A \cap B'$

Exactly one of A or B = $(A \cap B') \cup (B \cap A')$

$A' \cap B' = (A \cup B)'$

De Morgan's Laws

$$A' \cap B' = (A \cup B)'$$

$$A' \cup B' = (A \cap B)'$$

Relative Frequency

If $m(A)$ represents the number of times the event A occurs in M trials, then

$f_A = \frac{m(A)}{M}$ represents the relative frequence of occurrence of A on these trials.

If, for an event A, the limit of f_A as M approaches infinity exists, then one could assign prob. to A by

$$P(A) = \lim_{M \to \infty} f_A$$

This expresses the concept of statistical regularity.

When will this limit exist?

Motivate the def. of axioms of probability.

If S is a sample space, and $\underbrace{A, B, A_1, \dots, A_k, \dots}_{\text{pairwise mutually exclusive}}$ are events in S, then:

clearly $m(A) \geq 0$, $m(S) = M$

$\Rightarrow f_A \geq 0$

$m(A \cup B) = m(A) + m(B)$

$m(A_1 \cup A_2 \cup \dots \cup A_k \cup \dots) = \sum_{i=1}^{\infty} m(A_i)$

1) $f_A \geq 0$

2) $f_S = 1$

3) $f_{A_1 \cup A_2 \cup \dots} = f_{A_1} + f_{A_2} + \dots$