## 8.1 Operator Method

### Simultaneous Diff-eq

$$\boxed{\begin{aligned} \frac{d^2x}{dt^2} &= -5x + y \\ 2\frac{d^2y}{dt^2} &= 3x - y \end{aligned}}$$

or

$$\boxed{\begin{aligned} x' - 3x + y' + z' &= 5 \\ -y' + 2z' &= t^2 \\ x + y' - 6z' &= t - 1 \end{aligned}}$$

Solution:

A solution of the system is a set diff functions

$x = f(t)$

$y = g(t)$

$z = h(t)$ , and so on.

that satisfies each equation of the system on some interval I.

# Systematic Elimination

① $\dfrac{dx}{dt} = 2x - y \Rightarrow x' - 2x + y = 0$

$Dx - 2x + y = 0 \Rightarrow (D-2)x + y = 0$

$\dfrac{dy}{dt} = x \qquad y' - x = 0 \Rightarrow -x + Dy = 0$

$\boxed{\text{Eliminate the } y}$

Apply $-D$ to both sides

$$-D(D-2)x + -Dy = 0$$
$$-x + Dy = 0$$
$$-D(D-2)x - x = 0$$

$$D(D-2)x + x = 0$$
$$(D^2 - 2D + 1)x = 0$$
$$(D-1)^2 x = 0$$

$$x(t) = c_1 e^t + c_2 t e^t \quad (2)$$

Eliminate the $x$

$$(D-2)x + y = 0$$
$$-(D-2)x + D(D-2)y = 0$$

(Apply $(D-2)$ both side)

$$D(D-2)y + y = 0$$
$$(D^2 - 2D + 1)y = 0$$
$$(D-1)^2 y = 0$$
$$y = c_3 e^t + c_4 t e^t \quad (3)$$

(1)
$$(D-1)x + (D^2+1)y = 1$$
$$(D^2-1)x + (D+1)y = 2$$

$L_1 = D-1 \qquad L_2 = D^2+1 \qquad g_1 = 1$

$L_3 = D^2-1 \qquad L_4 = D+1 \qquad g_2 = 2$

$$\begin{vmatrix} D-1 & D^2+1 \\ D^2-1 & D+1 \end{vmatrix} x = \begin{vmatrix} 1 & D^2+1 \\ 2 & D+1 \end{vmatrix}$$

$$\left[(D-1)(D+1) - (D^2-1)(D^2+1)\right] x = (D+1)(1) - (D^2+1)(2)$$

$$\left[D^2 \cancel{-1} - D^4 \cancel{+1}\right] x = 0 + 1 - 2 = -1$$

$$(-D^4 + D^2)x = -1$$

$$(D^4 - D^2)x = 1$$

$$D^2(D^2-1)x = 1$$

(2) $$D^2(D-1)(D+1)x = 1$$

$$x_c = c_1 + c_2 x + c_3 e^{t} + c_4 e^{-t}$$

$$D^3(D-1)(D+1)x = 0$$

$$x = c_1 + c_2 x + c_3 x^2 + c_4 e^{t} + c_5 e^{-t}$$

$$x_p = c_3 x^2$$