$\boxed{2.1}$ First Order diffeq

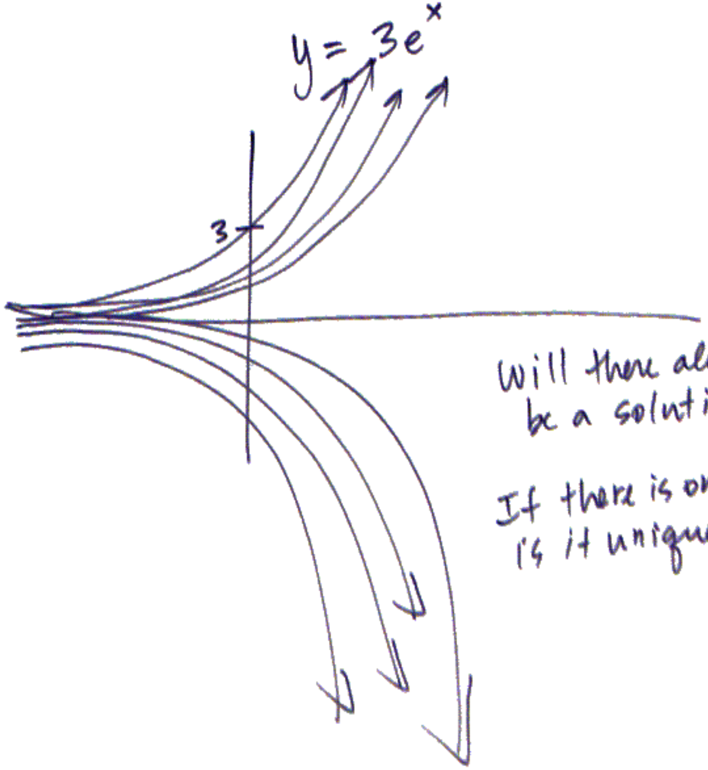
Preliminary Theory

$$\boxed{\begin{array}{l}\text{Initial Value prob (IVP)} \\ \dfrac{dy}{dx} = f(x,y) \quad \text{subject to} \quad \underbrace{y(x_0) = y_0}_{\text{initial cond.}} \\ \text{where } x_0 \in I,\ y_0 \in \mathbb{R}\end{array}}$$

Note: $y = ce^x$ is a solution to $y' = y$ on the interval $(-\infty, \infty)$

Note. If $y(0) = 3$, then

$3 = ce^0 \Rightarrow c = 3$



Will there always be a solution?

If there is one, is it unique?

Ex:

Note that the soln to

$\frac{dy}{dx} = xy^{1/2}$, $y(0) = 0$ has two solutions

$y = 0$ and $y = \frac{x^4}{16}$

Note: Hence, the solution is not always unique.

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why does this example not contradict the theorem on the right?

$$\frac{\partial f}{\partial y} = \frac{\partial (xy^{1/2})}{\partial y} = x \frac{1}{2\sqrt{y}}$$

^ not cont at $y = 0$.

Thm: Existence of a Unique Soln.

Let R be a rectangular region in the xy plane defined by $a \le x \le b$, $c \le y \le d$ that contains the point (x_0, y_0) in its interior.

If $f(x,y)$ and $\partial f / \partial y$ are continuous on R, then $\exists$ an interval I centered at x_0 and a $\exists!$ $y(x)$ defined on I satisfying the IVP.