9.4 Large Sample Properties.

DEF: Simple Consistency.

Let $\{T_n\}$ be a sequence of est. of $\tau(\theta)$. These est. are said to be consistent est. of $\tau(\theta)$ if, for every $\varepsilon > 0$,

$$\frac{\sum (x_i - \bar{x})^2}{n}$$

$$\lim_{n \to \infty} P\left[|T_n - \tau(\theta)| < \varepsilon \right] = 1$$

for all $\theta \in \Omega$

DEF: MSE Consistency.

$$\lim_{n \to \infty} E\left[T_n - \tau(\theta)\right]^2 = 0$$

$$\Rightarrow \lim_{n \to \infty} MSE(T_n) = 0$$

DEF: Asymptotically Unbiased.

T_n est. $\tau(\theta)$

$$\lim_{n \to \infty} E(T_n) = \tau(\theta)$$

Ex: $\hat{\theta}_n = \frac{n-1}{n} S^2$ $\quad$ $\hat{\theta}_n$ unbiased for σ^2? No!

$$E(\hat{\theta}_n) = \frac{n-1}{n} \sigma^2$$

Asymptotic Properties of MLEs

Under certain regularity conditions, the MLE $\hat{\theta}_n$ has the following prop.

1. $\hat{\theta}_n$ exists and is unique
2. $\hat{\theta}_n$ is a consistent est of θ.
3. $\hat{\theta}_n$ is asymptotically normal w/ asymptotic mean θ and variance $\dfrac{1}{nE\left(\frac{\partial}{\partial\theta}\ln f\right)^2} \Rightarrow \hat{\theta}_n \sim N(\theta, CRLB)$
4. $\hat{\theta}$ is asymptotically efficient.

Thm: A sequence $\{T_n\}$ of est of $\tau(\theta)$ is mean sequared consis. iff.

it is asymptotically unbiased and $\lim_{n\to\infty} Var(T_n) = 0$.

$\lim_{n\to\infty} b(T_n) = E(T_n) - \tau(\theta) = 0$

$MSE(T_n) = \underbrace{Var(T_n)}_{\to 0} + \underbrace{b(T_n)}_{\to 0}^2$

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Thm: If $\{T_n\}$ is simply consis for $\tau(\theta)$ and if $g(t)$ is continuous at each value of $\tau(\theta)$, then

$g(T_n)$ is simply consis. for $g(\tau(\theta))$

Ex: $\hat{\theta}_n = \left(\frac{n-1}{n}\right)s^2 \Rightarrow$ simply consistent?

$\sin(\hat{\theta}_n)$ simply consistent for $\sin(\sigma^2)$