$\boxed{8.2}$ Statistics.

DEF: A function of observable R.V.

$$T = t(X_1, \cdots, X_n)$$

which does not depend on any unknown parameters is called a statistic

Ex: Sample mean.

$$\bar{X} = t(X_1, X_2, \cdots, X_n) = \frac{1}{n}\sum X_i$$

Thm: If $X_1, \cdots, X_n$ denotes a R.V. from $f(x)$. with $E(X) = \mu$, $V(X) = \sigma^2$, then

$$E(\bar{X}) = \mu$$
$$V(\bar{X}) = \frac{\sigma^2}{n}$$

A statistic for which $E(\text{Stat}) = \text{parameter}$, we say the stat. is an <u>unbiased</u> estimate of the parameter

Ex: $Y \sim HYP(n, M, N)$

Suppose we want to estimate $\frac{M}{N} = p$.

Then
$$E\left[\frac{Y}{n}\right] = \frac{1}{n} E(Y) = \frac{1}{\cancel{n}} \frac{\cancel{n}M}{N} = \frac{M}{N}$$

$\frac{Y}{n}$ is an unbiased est. of $\frac{M}{N}$

$$Var\left(\frac{Y}{n}\right) = \frac{1}{n^2} Var(Y) = \frac{1}{n^2}\, n \frac{M}{N}\left(1 - \frac{M}{N}\right)\left(\frac{N-n}{N-1}\right)$$
$$= \frac{M}{N}\left(1-\frac{M}{N}\right)\left(\frac{1}{N-1}\right)\left(\frac{N}{n}-1\right)$$

$Var\left(\frac{Y}{n}\right) \to 0$ as $n \to \infty$.

Ex: Sample Variance.

$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$
$$= \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1}$$

S^2 is a function of the random sample.