7.2 Sequences of Random Variables.

Consider a sequence of random variables $Y_1, Y_2, \ldots$

with corresponding CDF's

$$G_1(y), G_2(y), \ldots, \ldots, \text{ where}$$

$$G_n(y) = P[Y_n \le y]$$

DEF: If $Y_n \sim G_n(y)$ for $n \in \mathbb{Z}^+$, and if some CDF $G(y)$

$$\lim_{n \to \infty} G_n(y) = G(y)$$

for all values y at which $G(y)$ is continuous, then the sequence $Y_1, Y_2, \ldots$ is said to converge in distribution to $Y \sim G(y)$

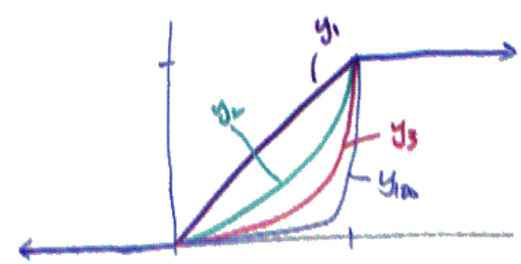
denoted by $Y_n \xrightarrow{d} Y$

The dist corresponding to $G(y)$ is called the limiting dist of Y_n.

EX: Let $X_1, X_2, \ldots, X_n$ be a R.S. from UNIF(0,1) $X_i \sim UNIF(0,1)$

The dist of the largest order statistic is

$$G_n(y) = \begin{cases} 0, & y \le 0 \\ y^n, & 0 < y < 1 \\ 1, & y \ge 1 \end{cases}$$



Note: $\lim_{n\to\infty} G_n(y) = \begin{cases} 0, & y<1 \\ 1, & y\geq 1 \end{cases}$

DEF: The function $G(y)$ of a degenerate distribution at the value $y=c$ if

$$G(y) = \begin{cases} 0, & y<c \\ 1, & y \geq c \end{cases}$$

In other words, $G(y)$ is the CDF of a discrete R.V. that assigns probability one at the value $y=c$ and zero otherwise.

$$g(y) = I_{\{c\}}(y)$$

Ex: The Idea of limiting dist is that we can use them to approx. probability. For example, rather than use the CDF of the actual dist, we can use the CDF of the limiting dist

Ex: A sample of size 10 is selected

$$F_T(t) = (1-e^{-t})^{10}, \ t>0$$

$$G(t) = \exp(-10e^{-t})$$

Then the following table shows the comparison

	1	2	5	7	9
$F_T(t)$	.0102	.2336	.9346	.9909	9988
$G(t)$	0253	.2584	.9348	9909	9988