

## 2.1 Intro to Random Variables (R.V)

**DEF 2.1.1** A random variable, say  $X$ , is a function defined over a sample space  $S$ , that associates a real number

$X(e) = x$ , with each possible outcome  $e$  in  $S$ .

↑            ↑  
capital    little

Capital letters indicate random variables  $(X, Y, Z)$   
lowercase letters indicate possible values the R.V can attain.

EX: Flip a coin 3 times

Sample space:  $S = \{$  HHH,  
THH  
HTH  
HHT  
TTH  
THT  
HTT  
TTT  $\}$

$X = \#$  of heads in 3 tosses.

$$X(\text{HHH}) = 3$$

$$X(\text{TTH}) = 1$$

$$X(\text{HTH}) = 2$$

$$X(\text{TTT}) = 0$$

EX: Roll 2 dice

1	2	3	4	5	6
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$X =$  maximum of 2 dice  $\rightarrow X(5,4) = 5$   
 $Y =$  sum of two dice  $\rightarrow Y(5,4) = 9$   
 $Z =$  min of the 2 dice

## 2.2 Discrete R.V.

DEF: If the set of all possible values of a R.V.  $X$ , is a countable set (Note: could be infinite)

$x_1, x_2, \dots, x_n$  or  $x_1, x_2, \dots$

then  $X$  is called a discrete R.V. The function

$$f(x) = P[X=x], \quad x = x_1, x_2, \dots$$

small      big      small

that assigns prob. to each possible  $x$  will be called the discrete prob. density function (pdf) (also called pmf = prob. mass function)

Sometimes we write  $f_x(x)$  instead of  $f(x)$

Thm 2.2.1

A function  $f(x)$  is a discrete pdf iff it satisfies both of the following for at most a countably infinite set of real #'s

1)  $f(x_i) \geq 0$  for all  $x_i$

2)  $\sum_{\text{all } x_i} f(x_i) = 1$

proof. follows from axioms of prob.

Homework!

ex. Flip a coin 3 times.

$$S = \{HHH, TTH, THT, HTT, THT, HTH, HHT, TTT\}$$

$X = \#$  of Heads in 3 tosses

$x$	$f(x)$
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

$x$	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

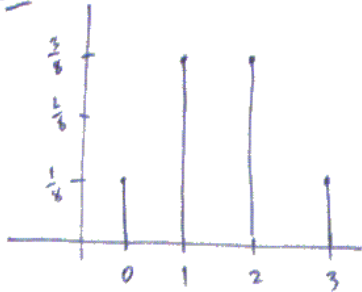
ex:  $X = \text{Max of two dice}$

$x$	$f(x)$
1	
2	
3	
4	
5	
6	

$x$	1	2	3	4	5	6
$f(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

## Graphical Representation

ex:



$$f(x) = {}_3 C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{3-x}$$

formulas:

You can also write  $f(x)$  as a formula:

$$f(x) = \frac{{}_3 C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{3-x}}{36}, x=1, 2, 3$$

