3.4 Cubic Spline Interpolation

- polynomials are oscillatory - they don't work well beyond the pts of support.

- what can we do?
  - piecewise poly. approx. -
    - series of piecewise straight lines / parabolas, etc.
    - not "smooth" -> we'll see jagged edges
  - we could use Hermite polynomials
    - for example, we could construct H_3 for each interval this would be smoother, but requires the derivative of f, which is not always available
  - Another choice, we can consider a technique of piecewise polynomial Interp, that requires no deriv info, except perhaps at the end pts of the intervals
    - simplest type => construct a quadratic on $[x_0, x_1]$ that agrees with f at x_0 & x_1

      => construct a second quadratic on $[x_1, x_2]$ that agrees with f at x_1 & x_2, etc.
  - since a quadratic needs three arbitrary const, $a_0 + a_1 x + a_2 x^2$, and only two conditions are required to fit the data, flexibility exists that allows the quadratic to be chosen so that, in addition, the interpolant has a continuous derivative on $[x_0, x_n]$.
  - The only difficulty is specifying conditions on the derivative at the end points. There isn't a sufficient number of constants to ensure the conditions will be met

The most comon type of piecewise polynomial approx using cubic polynomials between successive nodes is called cubic spline interpolation

- using cubic gives us the flexibility to ensure
  - interpolant is continuously diff.
  - " has a continuous second deriv.

  - It <u>does not</u> assume the derivatives of the interpolant match those of the function, even at the nodes.

<u>DEF</u>: Given f on $[a,b]$, and a set of nodes, $a = x_0 < x_1 < \cdots < x_n = b$

a cubic spline interpolant, S, for f is a function that satisfies the following conditions

a) S is a cubic polynomial, denoted S_j, on the subinterval $[x_j, x_{j+1}]$ for each $j = 0, \cdots, n-1$

b) $S(x_j) = f(x_j)$ $\quad j = 0, \cdots, n-2$

c) $S_{j+1}(x_{j+1}) = S_j(x_{j+1})$ $\quad$ "

d) $S'_{j+1}(x_{j+1}) = S'_j(x_{j+1})$ $\quad$ "

e) $S''_{j+1}(x_{j+1}) = S''_j(x_{j+1})$ $\quad$ "

f) One of the following is satisfied

(i) $S''(x_0) = S''(x_n) = 0$ (free or natural boundary)

(ii) $S'(x_0) = f'(x_0)$ and $S'(x_n) = f'(x_n)$ (clamped boundary

---

There are other more involved def for a spline, but this is sufficient to learn about,

Natural spline $\Rightarrow$ think of forcing a long flexible rod through all the nodes

clamped spline $\Rightarrow$ more accurate, but forces you to know more info. about the derivs. at the endpts. (or an approx)