Math 334

Hypothetical Exam 2, Fa2186, (Chapter 4, 5 in Zill, 5th Ed.)
October -7, 2186
S. K. Hyde Name:_____
Show all your work to receive credit. All answers must be justified to get full credit.

These questions are intended to give you some idea of the types of questions which could be asked on your exam. They may not cover all of the topics which will be on your exam (and they may cover more topics than are on your exam). The length of your exam may be shorter than this practice exam (or longer). Working these problems is not a substitute for studying your notes and reading the book.

True or False

Circle T or F corresponding to the best answer. (2 pts each)

1. **T F** Two solutions y_1 and y_2 of $y'' + y' + y = 0$ are linearly dependent if $W(y_1, y_2) \neq 0$ for one value of x on an interval.
2. **T F** Three non-zero functions f_1, f_2, and f_3 are linearly independent if you can write one of them as a linear combination of the others.
3. **T F** A constant multiple of a solution of a differential equation is also a solution to the same differential equation.

Fill in the Blank

Write the best answer in the spaces provided

4. (3 pts each) The functions $f_1(x) = x^2$ and $f_2(x) = x|x|$ are linearly independent on the interval _____, whereas they are linearly dependent on the interval _____.
5. (3 pts each) Find the lowest order linear differential operator that annihilates the following:

   (a) $7 + x^3 e^{2x} + x^3$

   (b) $x^5 e^{-87} \cos 9x$

   (c) $e^{4x} \cos 3x$

   (d) $(1 + x + x^4) \sin x$

   (e) $\cosh 2x$

   (f) $\sin^2 x$

   (g) $\sin x \cos x$

Show Your Work
Show all work clearly and neatly. No work shown means no credit will be given. Use correct notation to get full credit. Reserve scratch paper work for scratch paper, which means only include necessary work on the exam. Erase all mistakes neatly. Keep it neat!

1. (10 pts) Determine whether $f_1(x) = 3x^2$, $f_2(x) = x - 2x^2$, and $f_3(x) = 7x$ are linearly independent or dependent on $(-\infty, \infty)$. If they are dependent, show their dependence by providing the constants c_1, c_2, and c_3. Choose only constants c_1, c_2, and c_3 that are integers, and the sum of the squares of the integer constants is the smallest possible.

2. (10 pts) Form the general solution to the differential equation

$$(1+x)y'' + xy' - y = 0,$$

given that $y_1 = e^{-x}$ is a solution to the differential equation.

3. (10 pts) Find all the linearly independent functions that are annihilated by $D(D-3)^2(D^2-4D+20)$.

4. (10 pts) Solve the differential equation

$$3y''' - 8y'' - 4y' + 16y = 0,$$

subject to the initial conditions $y(0) = y'(0) = 0$, and $y''(0) = 3$.

5. (10 pts) Determine the form of a particular solution for

$$y''' - 6y'' + 9y' = 1 + 5x^2 + 3e^x + 4x^2e^{3x}.$$

Note that you are not to solve for the coefficients, just find the form.

6. (10 pts) Evaluate: $D(D+2)(1 + 2x + \cos x + e^{3x})$

7. (10 pts) Solve: $y''' - 4y'' + 5y' - 2y = 2xe^x - 1$.

8. (10 pts) Solve the differential equation $y'' + y = \sin x$ by using variation of parameters. Simplify completely.

9. A 10-lb weight attached to a spring stretches it 2 ft. The weight is attached to a dashpot damping device that offers a resistance numerically equal to β ($\beta > 0$) times the instantaneous velocity. Determine the values of the damping constant β so that the subsequent motion is

   (a) (3 pts) overdamped

   (b) (3 pts) critically damped

   (c) (3 pts) underdamped

10. (10 pts Extra Credit) Find the lowest ordered differential operator that annihilates $\sin^7 x$.

11. (3 pts) How many languages do you speak fluently?

12. (3 pts) After you get married, how many kids do you want to have?

13. (3 pts) Describe how you have created and submitted homework to Canvas. What could be improved on it? Any suggestions?