Math 221: Inferences for Two Means: Independent Samples
S. K. Hyde
Chapter 18 (Moore, 5th Ed.)

Test of Significance for the Difference of Two Means ($\mu_1 - \mu_2$)

A study comparing attitudes toward death was conducted in which organ donors (individuals who had signed organ donor cards) were compared with non-donors. The study is reported in the journal *Death Studies* (Vol. 14, No. 3, 1990). Templer’s Death Anxiety Scale (DAS) was administered to both groups. On this scale, high scores indicate high anxiety concerning death. These results were reported below. Test the claim that organ donors have lower DAS scores than nonorgan donors.

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organ Donors</td>
<td>25</td>
<td>5.36</td>
<td>2.91</td>
</tr>
<tr>
<td>Nonorgan Donors</td>
<td>69</td>
<td>7.62</td>
<td>3.45</td>
</tr>
</tbody>
</table>

Table 1: Templer’s Death Anxiety Scale

The following notation is used:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}_1$</td>
<td>mean of sample 1</td>
</tr>
<tr>
<td>$\bar{x}_2$</td>
<td>mean of sample 2</td>
</tr>
<tr>
<td>$s_1$</td>
<td>standard deviation of sample 1</td>
</tr>
<tr>
<td>$s_2$</td>
<td>standard deviation of sample 2</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>mean of population 1</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>mean of population 2</td>
</tr>
<tr>
<td>$n_1$</td>
<td>size of sample 1</td>
</tr>
<tr>
<td>$n_2$</td>
<td>size of sample 2</td>
</tr>
</tbody>
</table>

$H_0$:  

$H_a$:  

$\alpha$:  

Test Statistic:  

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$P$-value:  

$(df = \min(n_1 - 1, n_2 - 1))$

Decision: (Circle one:) Reject $H_0$ or Fail to Reject $H_0$

Conclusion:
Confidence Interval for $\mu_1 - \mu_2$

The confidence interval estimate of the difference $\mu_1 - \mu_2$ is:

$$(\bar{x}_1 - \bar{x}_2) \pm m$$

where

$$m = t^* SE \quad \text{and} \quad SE = \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}$$

and $df = \min(n_1 - 1, n_2 - 1)$.

Construct a 95% confidence interval estimate for the difference between the mean DAS scores of organ donors versus non-organ donors.

$$m = t^* \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}} =$$

So a 95% confidence interval estimate for the difference between the mean DAS scores of organ donors versus non-organ donors is

$$(\bar{x}_1 - \bar{x}_2) \pm m$$
Solution:

<table>
<thead>
<tr>
<th>$H_0$: $\mu_1 - \mu_2 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_a$: $\mu_1 - \mu_2 &lt; 0$</td>
</tr>
<tr>
<td>$\alpha = 0.05$</td>
</tr>
</tbody>
</table>

**Test Statistic:**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(5.36 - 7.62) - (0)}{\sqrt{\frac{2.91^2}{25} + \frac{3.45^2}{69}}} = -3.1608$$

**P-value:**

- $P(T < -3.1608) = .0013347$ (exact from calculator)
- $P(T < -3.161) < .005$ (df = 24) (Table)

**Decision:** Since $P$-value < .05, Reject $H_0$

**Conclusion:**

There is sufficient evidence that organ donors have lower DAS scores than nonorgan donors.

Note: When using a TI-83 or TI-84 calculator, select $\text{STAT} \rightarrow \text{TESTS} \rightarrow \text{2SampTTest}$ and enter the appropriate data or statistics. Note that when using a computer or a calculator, the degrees of freedom are more exact, rather than the conservative degrees of freedom ($df = \min(n_1 - 1, n_2 - 1)$). The calculator uses the degrees of freedom of

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{2.91^2}{25} + \frac{3.45^2}{69}\right)^2}{\frac{1}{24}\left(\frac{2.91^2}{25}\right)^2 + \frac{1}{68}\left(\frac{3.45^2}{69}\right)^2} = 50.08457$$

Note that this calculation is automated by the calculator. If you don’t have a TI-83 or TI-84, you should use the conservative degrees of freedom.
Solution:

Confidence Interval for $\mu_1 - \mu_2$

Construct a 95% confidence interval estimate for the difference between the mean DAS scores of organ donors versus non-organ donors.

First, using $df = \min(n_1 - 1, n_2 - 1)$,

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(2.91)^2}{25} + \frac{(3.45)^2}{69}} = .7149993$$

and

$$m = t^* SE = 2.064(.7149993) = 1.4758$$

So a 95% confidence interval estimate for the difference between the mean DAS scores of organ donors versus non-organ donors is

$$(\bar{x}_1 - \bar{x}_2) \pm m$$

$$-2.26 \pm 1.4758$$

$$(-3.7358, -.784241)$$

Note: When using a TI-83 or TI-84 calculator, select $\text{STAT} \rightarrow \text{TESTS} \rightarrow \text{2SampTInterval}$ and enter the appropriate data or statistics. When using a calculator, the degrees of freedom are different (but gives a more accurate confidence interval). In this case, the degrees of freedom are $df = 50.08$ and the margin of error is

$$m = t^* SE = 2.008(.7149993) = 1.43572$$

So a 95% confidence interval estimate for the difference between the mean DAS scores of organ donors versus non-organ donors is

$$(\bar{x}_1 - \bar{x}_2) \pm m$$

$$-2.26 \pm 1.43572$$

$$(-3.6957, -.8243)$$