Show all your work to receive credit. All answers must be justified to get full credit.

These questions are intended to give students in Math 112 some idea of the types of questions which could be asked on an exam. The questions may not cover all of the topics which will be on your exam (and they may cover more topics than are on your exam). The length of your exam may be shorter than this practice exam. Working these problems is not a substitute for studying your notes, reading the book, or doing homework problems.

1. Find a formula for the \( n^{\text{th}} \) term of the following sequences. Start each sequence at \( n = 1 \).

(a) (10 pts) \[1, -4, 9, -16, 25, -36, \cdots\]

(b) (10 pts) \[1, 5, 9, 13, 17, \cdots\]

(c) (10 pts) \[1, 0, 1, 0, 1, 0, \cdots\]

(d) (10 pts) \[0, 3, 8, 15, 24, \cdots\]

(e) (10 pts) \[1, 1, 2, 3, 5, 8, 13, 21, 34, \cdots\]
2. Determine whether the following sequences converge. If they converge, find the limit.

(a) (10 pts) \( a_n = \frac{\sin^2 n}{n^2} \)

(b) (10 pts) \( a_n = \sqrt[3]{n^2} \)
(c) (10 pts) $a_n = (n + 4)^{1/(n+4)}$

(d) (10 pts) $a_n = \sqrt[3]{4^n n}$
(e) (10 pts) \( a_n = \left( \frac{n + 1}{2n} \right) \left( 1 - \frac{1}{n} \right) \)

(f) (10 pts) \( a_n = \tanh n \)
3. Determine the sum of the following convergent series. If it doesn’t converge, state why.

(a) (10 pts) \[ \sum_{n=2}^{\infty} e^{-3n} \]

(b) (10 pts) \[ \sum_{k=3}^{\infty} \frac{3^k}{k^{k-1}} \]
(c) (10 pts) \[ \sum_{k=2}^{\infty} \frac{1}{k(k+1)} \]

(d) (10 pts) \[ \sum_{k=0}^{\infty} \frac{e^{n\pi}}{\pi n e} \]
4. Determine whether the following series converge or diverge.

(a) (10 pts) \( \sum_{k=1}^{\infty} \frac{k \ln k}{3^k} \)

(b) (10 pts) \( \sum_{n=1}^{\infty} \frac{1}{n(1 + \ln^2 n)} \)
5. Determine whether the series are absolutely convergent, conditionally convergent, or divergent.

(a) (10 pts) \( \sum_{k=1}^{\infty} (-1)^k \sqrt[6]{10} \)
(b) (10 pts) \[ \sum_{k=1}^{\infty} \frac{(-1)^k (k!)^2 4^k}{(3k + 1)!} \]

(c) (10 pts) \[ \sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{n^2} \]
6. (10 pts) Find the radius and interval of convergence for the following power series.

\[
\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3x + 1)^n}{n}
\]
7. (10 pts) Find a power series representation for \( \int_0^x \cos(t^2) \, dt \).
8. (10 pts) Find the Maclaurin Series representation for $x \cos(\pi x)$.
9. (10 pts) Find the Taylor series for \( f(x) = \frac{1}{x-3} \) about \( x = 1 \).
f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos \left( \frac{n\pi x}{L} \right) + b_n \sin \left( \frac{n\pi x}{L} \right) \right]

a_0 = \frac{1}{L} \int_{-L}^{L} f(x) \, dx \quad a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \left( \frac{n\pi x}{L} \right) \, dx \quad b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \left( \frac{n\pi x}{L} \right) \, dx

10. (10 pts) Find the Fourier series for \( f(x) = \begin{cases} 0, & 0 < x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x < \pi \end{cases} \). The formulas you need are given above. Choose \( L = \pi \).