Show all your work to receive credit. All answers must be justified to get full credit.

These questions are intended to give students in Math 112 some idea of the types of questions which could be asked on an exam. The questions may not cover all of the topics which will be on your exam (and they may cover more topics than are on your exam). The length of your exam may be shorter than this practice exam. Working these problems is not a substitute for studying your notes, reading the book, or doing homework problems.

Show Your Work
Show all work clearly and neatly. No work shown means no credit will be given. Use correct notation to get full credit. Reserve scratch paper work for scratch paper, which means only include necessary work on the exam. Erase all mistakes neatly. Keep it neat!

1. Given that $\sinh x = \frac{12}{5}$, $x > 0$, then find the following: (You may use $\cosh^2 x - \sinh^2 x = 1$ without proof).

<table>
<thead>
<tr>
<th>Function</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\cosh x$</td>
<td></td>
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<tr>
<td>$\tanh x$</td>
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<tr>
<td>$\sech x$</td>
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<td>$\csch x$</td>
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<tr>
<td>$\coth x$</td>
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2. Determine the derivative of $y$ with respect to the independent variable:

(a) $y = t \ln(t)$

(b) $y = \frac{1 + \ln t}{t}$
(c) $y = (9x^2 - 6x + 2)e^{3x}$

(d) $y = 3 \log_8 (\log_2 t)$
(e) \( y = (\ln x)^x \)

(f) \( y = \tan^{-1} (\ln x) \)
(g) \( y = \ln \left( \frac{e^x}{1 + e^x} \right) \)

(h) \( y = \ln(\sinh 2z) \)
(i) \( y = \ln (\ln (\ln x)) \)

3. Find the derivative of \( y = \sqrt{\frac{(x+3)(x+4)(x+5)}{x(x+3)}} \) using logarithmic differentiation.
4. Evaluate the following integrals:

(a) \[ \int e^{\csc(\pi + t)} \csc(\pi + t) \cot(\pi + t) \, dt \]

(b) \[ \int \frac{2y}{y^2 - 25} \, dy \]
(c) \[ \int \frac{dx}{x^2} \]

(d) \[ \int \frac{dy}{y^2 - 4y + 5} \]
5. Show that \( \tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x} \), where \(|x| < 1\).
6. Verify the identity $2 \cosh^2 x = \cosh 2x + 1$
7. Does \( f \) grow faster, slower, or at the same rate as \( g \) as \( x \to \infty \)? Give reasons for your answers.

(a) \( f(x) = \ln 2x, g(x) = \ln x^2 \).

(b) \( f(x) = x, g(x) = \tan^{-1} x \).
8. Find the limit of the following

(a) \( \lim_{x \to 0^+} \left( 1 + \frac{3}{x} \right)^x \).

(b) \( \lim_{x \to 0} \frac{2^{-\sin x} - 1}{e^x - 1} \).
9. What costs $27 million per gram and can be used to treat brain cancer, analyze coal for its sulfur content, and detect explosives in luggage? The answer is californium-252, a radioactive isotope so rare that only 8g of it have been made in the western world since its discovery by Glenn Seaborg in 1950. The half-life of the isotope is 2.645 years – long enough for a useful service life and short enough to have a high radioactivity per unit mass. One microgram of the isotope releases 170 million neutrons per second.

(a) What is the value for $k$ in the decay equation for this isotope?

(b) How long will it take 95% of a sample’s radioactive nuclei to disintegrate?

(c) How long will it take 99.9% of a sample’s radioactive nuclei to disintegrate? Is it substantially different than part (b)?
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