Math 112

Hypothetical Exam 3, (Chapter 4 in Thomas, Weir, Hass, and Giordano, 11th Ed.)
October 16, 1793
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Show all your work to receive credit. All answers must be justified to get full credit.

These questions are intended to give students in Math 112 some idea of the types of questions which could be asked on an exam. The questions may not cover all of the topics which will be on your exam (and they may cover more topics than are on your exam). The length of your exam may be shorter than this practice exam. **Working these problems is not** a substitute for studying your notes, reading the book, or doing homework problems.

Show Your Work

Show all work clearly and neatly. No work shown means no credit will be given. Use correct notation to get full credit. Reserve scratch paper work for scratch paper, which means only include necessary work on the exam. Erase all mistakes neatly. Keep it neat!

1. (5 pts) Suppose that $f'(x) > 0$ and $f''(x) > 0$ on (a, b). Which of the following is correct?

   A. $f(x)$ is increasing and concave up.
   B. $f(x)$ is decreasing and concave up.
   C. $f(x)$ is increasing and concave down.
   D. $f(x)$ is decreasing and concave down.
   E. There is not enough information to make a conclusion about $f(x)$ on (a, b).

2. (10 pts) Find all possible functions with the given derivative of

$$y' = \sin(2t) + \cos\left(\frac{t}{2}\right)$$

3. (10 pts) Use the Mean Value Theorem to determine c for $f(x) = x^3 + 2x^2 - 3x + 5$ over the interval $[0, 2]$.

4. (10 pts) If b, c, and d are constants, for what value of b will the curve $y = x^3 + bx^2 + cx + d$ have a point of inflection at $x = 1$? Give reasons for your answer.

5. (10 pts) Find the intervals on which the function $h(x) = 2x^3 - 18x$ are increasing and decreasing.

6. (10 pts) Find $\lim_{t \to 0} \dfrac{10(\sin t - t)}{t^3}$ using l'Hôpital's Rule.

7. (10 pts) Suppose the derivative of the function $y = f(x)$ is

$$y' = (x-1)^2(x-2)(x-4)$$

At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection?

8. (10 pts) A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.

9. The method for finding the third root of a real number a is

$$x_{n+1} = \frac{1}{3}\left(\frac{a}{x_n^2} + 2x_n\right). \tag{1}$$

(a) (10 pts) Form this method by applying Newton's Method to $f(x) = x^3 - a$.

(b) (5 pts) Use the method in equation (1) above to find the second approximation (x_2) to the third root of 5. Start with an initial guess of $x_0 = 1$.

10. (10 pts each) Find the following indefinite integrals:

(a) $\displaystyle\int \frac{6}{(r-3)^3}\,dr$

(b) $\displaystyle\int -3\csc^2 x\; dx$

(c) $\int 5 - 6x \; dx$

(d) $\int \tan^2 x \; dx$

11. (10 pts) Find the function y that satisfies the differential equation

$$\frac{d^3y}{dx^3} = 6,\ y''(0) = -8,\ y'(0) = 0,\ y(0) = 5.$$